2016

# NUMBER SENSE IN HIGH SCHOOL MATHEMATICS STUDENTS

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by

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Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN MATHEMATICS

THE UNIVERSITY OF TEXAS AT ARLINGTON

May 2016

Acknowledgements

I would like to express my special thanks to my professor, Dr. Jorgensen who gave me all the help and support I needed on this project.

I also want to thank everyone in my family for their inspiration and encouragement throughout my graduate career.

December 17, 2015

# Abstract

## NUMBER SENSE IN HIGH SCHOOL MATHEMATICS STUDENTS



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Understanding the real number system plays a very important role in each student's mathematical achievement. The Texas Essential Knowledge and Skills (TEKS) for Mathematics Subchapter A. Elementary states, "For students to become fluent in mathematics, students must develop a robust sense of number" (TEKS Subchapter A Elementary, 2012). Knowledge of the real number system and number sense develops over several years. Once students get to high school, they are expected to have a large amount of knowledge about the real number system and number sense in order to effectively start and complete their high school math courses. However, many high school students struggle with real number concepts and operations. The purpose of this project is to investigate the area(s) of number sense that high school students need to understand in order to be successful in mathematics.

A number sense assessment tool was developed specific to students at the secondary level. The tool was used to evaluate the number sense of 124 high school students in varied mathematics courses. The outcomes of the number sense assessment were compared with the students' most recent standardized math score, as well as the grade of the first quarter of the highest common level high school math class. The result shows a positive correlation between secondary students' number sense knowledge and their mathematics ability.

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## Chapter 1

## Number Sense is Important

Number sense, number knowledge or the ability to work with numbers is so critical to students' success in mathematics that many people have tried to explain and define it. One definition provided in the book *Helping Children Learn Mathematics* explains what number sense means: "Number sense, like common sense is difficult to define or express simply. It refers to an intuitive feel for numbers and their various uses and interpretations. Number sense also includes the ability to compute accurately and efficiently, to detect errors, and to recognize results as reasonable. People with number sense are able to understand numbers and use them effectively in everyday living. Good number sense is also recognizing the relative magnitudes of numbers and establishing referents, or benchmarks, for measures of common objects and situations in their environments" (Reys, 2004, p. 138). This definition also gives the elements and concepts of number sense that students need to know. According to this definition, having a good knowledge of number, or having number sense, not only means being able to do number computation, but students must also be able to recognize number magnitude, identify relationship between different sets of numbers, and tell apart reasonable solution(s) to real life application(s).

There is no doubt that the knowledge of numbers plays a very important role in each student's mathematics career. The Texas Essential Knowledge and Skills (TEKS) for Mathematics Subchapter A. Elementary states, "For students to become fluent in mathematics, students must develop a robust sense of number" (Texas Education Agency [TEA], 2012). Number sense is an emerging construct that refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean, an ability to perform mental mathematics and to look at the world and make comparisons (Chard, Gersten, 1999). Many recent studies have linked students' mathematics skills and problem solving skills to number sense. Louange and Bana (2010) conducted a study on seventh grade students to determine the relationship between students' number sense and their problems solving skills. The data was obtained by classroom observations, interviews given to students and teachers, as well as a paper and pencil test. The results showed that problem-solving performance depends upon number sense proficiency. Another study done by Jordan, Kaplan, Locuniak, and Ramineni (2007) indicated that screening early number sense development is useful for identifying children who will face later math difficulties or disabilities.

Knowledge of number sense has also been linked to high levels of mathematics education. According to Castronovo and Göbel (2012), there are two different number systems; the Approximate Number system (ANS) relates to the rough estimation of quantity, and the Exact Number System (ENS) tends toward the exact calculation of numbers. The study done by Castronovo and Göbel (2012) points out that high level mathtematics education in adults is associated with the ENS, which develops later in life following the acquisition of symbolic number knowledge. Therefore, the level of understanding that students have with number can be an indicator of how well and how far students will advance in high-level mathematics classes (Castronovo, Göbel, 2012). When it comes to number sense, "higher level conceptual structures depend on the core concepts that student(s) acquire at a younger age" (Griffin, 2004). Dr. Griffin also states that "students whose core structure of number knowledge is not in place at the expected age will experience serious delays and will have difficulty catching up with their peers" (Griffin, 2004). Moreover, many students who have a poor sense of numbers in kindergarten often show evidence of mathematics disabilities later in life (Mazzocco, Thompson, 2005). Also, students who were able to recognize relative magnitudes of numbers which was called Quantity Discrimination Measure (QD) in a study done by Clarke and Shinn (2004) usually are the ones that have higher mathematics achievement. Thus, students who have better number sense will advance higher and be more successful in mathematics classes; this supports the argument that the knowledge of numbers lays the foundation for students' later success in mathematics (Castronovo, Göbel, 2012).

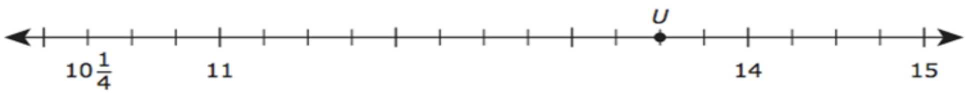
The better students are at utilizing and working with real numbers, the greater their mathematical achievement (Clarke, Shinn, 2004). Students' knowledge of the real number system and number sense develops over several years (TEA, 2012). Furthermore, students must master the basic concepts before they can move on to more complicated ones. "Behr and Post (1988) suggested that children needed to be competent in the four operations of whole numbers, along with an understanding of measurement, for them to understand rational numbers (as cited in Pearn, 2007, p.31). Moreover, in an article published in *Teaching Mathematics through Problem Solving*, Hiebert and Wearne (2003) proposed that a student who understands the meaning of adding or subtracting simple rational numbers will be able to apply the same concept to add and subtract rational expressions (p.4).

In Texas, the real number system and number sense are taught to students from pre-kindergarten to twelfth grade with topics varying from very rudimentary to complex. The counting numbers (or natural numbers) are introduced in pre-kindergarten and kindergarten; students are expected to learn the operations (addition, subtracting, multiplication, and division) on counting numbers and zero from first grade to third grade; simple fractions and decimals (rational numbers) are introduced at the end of third grade and by the end of fifth grade students must be able to work well with rational numbers (Texas Essential Knowledge and Skills for Mathematics [TEKS] Subchapter A.

Elementary, 2012). When students get to middle school, they will continue to review and enrich their understanding of rational numbers; at the same time, students will begin to learn about integers in sixth grade and seventh grade. At eighth grade, which is the end of middle school, irrational numbers will be taught to student via square roots of natural numbers and the Pythagorean Theorem (TEKS Subchapter B. Middle School, 2012). When first arriving to high school, a freshman is expected to have a strong number sense and thorough knowledge of the real number system in order to effectively start and complete his or her high school math courses.

Since number sense plays a crucial role to students' mathematics abilities, it is one of the main foci of state testing; in Texas, students will start statewide standardized testing in third grade; mathematics is a subject in which students have to test every year from third grade to ninth grade. Starting from the third grade, students are tested on the topics of number relationships, number magnitude, operations involving numbers and referents for numbers. However, for these third grade students, their Mathematics State of Texas Assessments of Academic Readiness (STAAR) test is based only on natural and whole numbers (TEKS Subchapter A. Elementary, 2012)); other types of numbers are introduced as students advance to higher grade levels. Even though there are different sets of numbers in the real number system, the basic concepts to understand them remain the same. In the example below from the 2014 fourth grade mathematics STAAR test, students were asked to use number magnitude to find the location of a rational number.

**2** Which number does point *U* represent on the number line below?




**F** $14\frac{2}{4}$

**G** $13\frac{3}{4}$

**H** $12\frac{2}{4}$

**J** $13\frac{2}{4}$

Figure 1-1 Using magnitude to locate rational numbers, State of Texas Assessments of Academic Readiness (STAAR) test 4th grade 2014 © Texas Education Agency (TEA)[1]

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The same type of problem appeared in the 2014 third grade STAAR Mathematics test. Students were asked to locate a natural number on the given number line. Students need to use the numbers' magnitudes for this problem; however, in the fourth grade question, the numbers used were rational. In this third grade question, natural numbers were used.

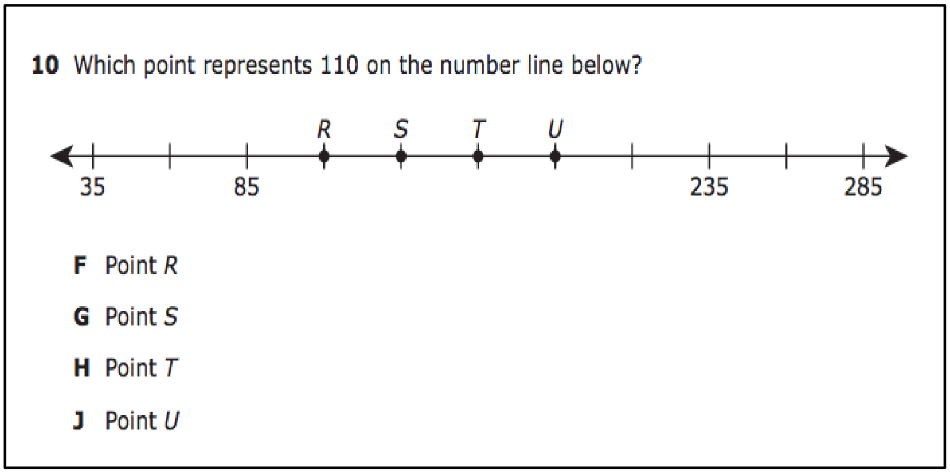



Figure 1-2 Using magnitude to locate natural numbers, STAAR test 3rd grade 2014 © TEA[1]

Most of the students who struggle with math in high school, have been struggling with math since they were in elementary school (Most Ontario students can read, 2014); and number sense is one of the main pieces of information students need to develop in elementary school. Moreover, numbers are fundamental to concepts in mathematics. Thus, learning about the number system and having number sense are key to students' successes in mathematics from kindergarten to twelfth grade.

In high school, the presence of number sense is subtler, but its existence is undeniable. Students will need to know number magnitude when it comes to plotting points on the coordinate plane, graphing solution sets for inequalities, graphing graphs for different functions and much more.

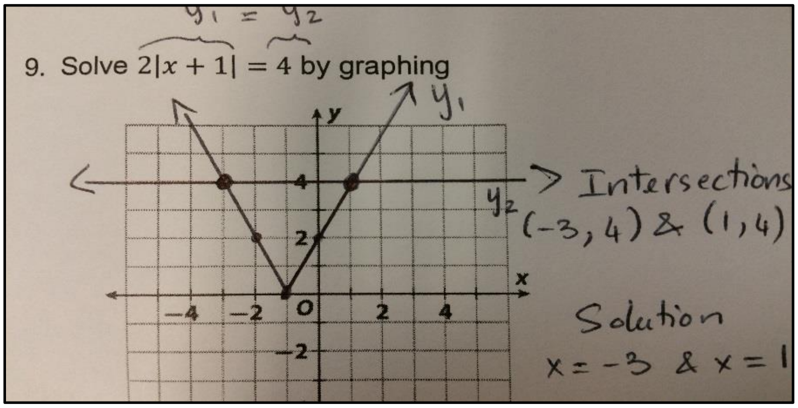



Figure 1-3 Using number magnitude to graph and solve absolute value equation

High school students also need to be able to do computation with numbers from different sets such as, natural numbers, whole numbers, integers, rational numbers, and irrational numbers in order to solve equations and evaluate expressions effectively. Number discrimination is the one area of number sense that students sometimes struggle with; using number discrimination, students must be able to identify whether or not the solution makes sense for each problem. For instance, students must know that only whole numbers can be used to describe pets, humans, cars, etc.; whereas, when it comes to measurements such as speed, then students must know that they can use any real number.

Lacking number sense and the knowledge of the real number system does not only affect students at the high school level. A recent research study was conducted by Dr. Geary at the University of Missouri on 180 thirteen-year-old students. The study found that students who fall behind their peers in a test of basic math skills needed for a regular adult job, were the same ones who had struggled with number sense when they were in first grade (Geary, Horad, Nugent, Bailey, 2013). The students who struggle with number sense are more likely to have problem(s) in other math classes. These students are also prone to have problems later on if they are holding jobs that require math skills (Geary et al., 2013).

On the whole, understanding the real number system and having a good sense of numbers is important to each student's mathematical career. The purpose of this project is to investigate how the understanding of numbers and number sense play out in students' in high school mathematical experiences.

# Chapter 2

## Developing a Tool to Assess High School Students' Understanding of the Real Number System

In recent years, McGraw Hill Education has published a program called *Number World*, an intervention program that helps pre-kindergarten through eighth grade students to be up to grade level on number sense. In this program students are given a placement test; the results of the placement test are used to determine the level of knowledge that students have about numbers. Depending on the levels, students are then taught different lessons varying from remediation to acceleration. This placement is call the Number Knowledge Test; the *Number World* program and the test were developed by Sharon Griffin, a professor at Clark University in Massachusetts. (Guthrie, 2014)

The assessment used in this project was created using the same format as the Number Knowledge Test; it is used to assess the student on the levels of understanding that students have about the real numbers system and number sense. The assessment used in this project has three different levels of understanding: basic (level one), intermediate (level two) and advanced (level three). The basic level questions correspond to the TEKS at the elementary level (kindergarten through fifth grade); the intermediate questions correspond to the middle school TEKS (grade six through eight); and the advanced questions correspond to high school Algebra I, Algebra II and Geometry TEKS. There are ten questions for each level which will be vertically aligned throughout the assessment.

The source for the first level questions and some of the level two questions will be pulled from released mathematics STAAR tests from third grade to eight grade in the last two years. Some of the questions for the third level or second level of the assessment will be taken from The Kalamazoo Area Algebra Project (KAAP) and the End of Course (EOC) exam. The KAAP is a program developed as a joint effort between Western Michigan University, the Kalamazoo Area Mathematics and Science Center, and southwest Michigan schools; the purpose of the project was to aid 6th to 12th grade math teachers in teaching algebra and pre-algebra (Western Michigan University, 2010-2012). The KAAP had a very large scope, but only the questions from the real number system module will be used in this assessment.

1. **Classify the following numbers as rational or irrational.**

   (a) $\sqrt{3} - 1$

   (b) 0.25312531...

   (c) 3.1416

   (d) 26.12131415...

   (e) $-5/13$

   (f) $\pi$

   (g) $\sqrt{121}$

Figure 2-1 Example question from the Kalamazoo Area Algebra Project, real-number module[2]

The first element of number sense and the real number system that will be assessed is the ability to locate a number given on the number line. Students must understand that each number represents a measurement or a magnitude; thus, student must consider the magnitude of numbers shown on the number line in order to find the correct location for the given number. As listed in TEKS 2C of third grade mathematics, students are expected to represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 (TEKS Subchapter A. Elementary, 2012); meanwhile, sixth grade students are expected to locate, compare, and order integers and rational numbers using a number line (grade 6 TEKS 2C, 2012); and by the end of middle school, a student is expected to do the same with irrational number (grade 8 TEKS 2B, 2012); on the other hand, a high school student is required to be able to locate intervals of number on number lines; according to Algebra I TEKS 2A and 6A, a student must be able to identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete (TEKS Subchapter C, 2012); When the function is continuous, the domain and range are not just a few numbers; domain and range of continuous functions are represented by intervals; Algebra I students have to look at any given graph and then be able infer from that graph the intervals of real numbers on the x-axis for domain and y-axis for range. To portray that each given function graph has two dimensions, the x-axis and y-axis are constructed using two number lines intersecting at the origin (0, 0).

Another area in high school mathematics that may require students to have a strong knowledge of number magnitude is absolute value functions, equations and inequalities as listed under Algebra 2 TEKS 4A. Students who have a good knowledge of number and number sense will have a good foundation on which to understand that absolute value means the distance from a number to zero; thus, seeing 11 and -1 as solutions to the equation $|x - 5| = 6$ is reasonable to the student. However, students who are weaker in their understanding of the real number system will not see that -1 is also an answer. Moreover, students again will see the component of number magnitude in many geometry concepts. Geometry TEKS 2 B requires students to derive and use the distance, slope, and midpoint formulas to verify geometric relationships. In order to figure out the midpoint, a student has to divide the segment into two equal parts with the same magnitude.

Understanding numbers' magnitudes is very important in mathematics; according to Valerie Faulkner, “virtually all mathematical topics can be modeled for students using quantity/ magnitude as a core communicator" (Faulkner, 2009). The ability to locate a number on a given number line will be reflected on the first two questions of each level of the assessment. For the first two problems in the basic level, students were asked to place the natural number in the correct place, given the real number line. These two questions were both created using the same format as problems from a third grade mathematics STAAR test in 2014.

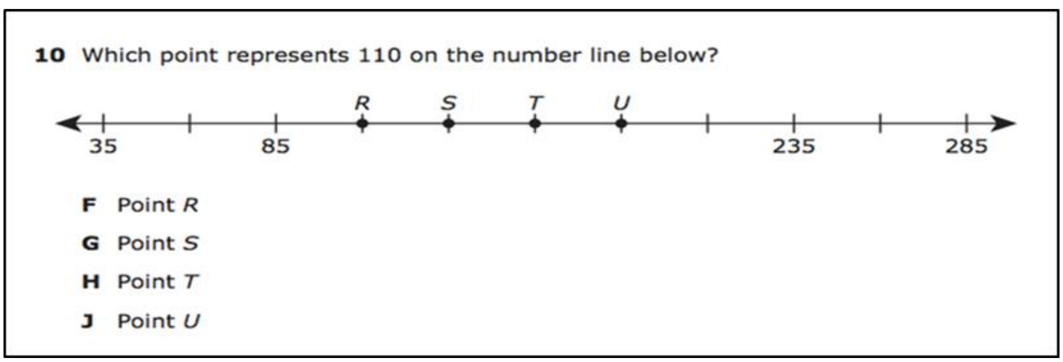
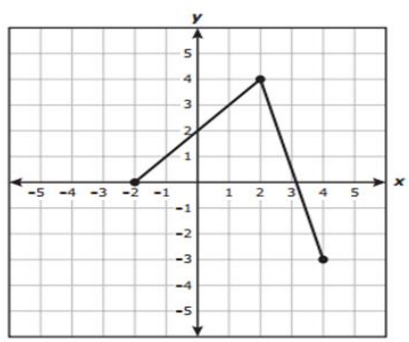



Figure 2-2 Represent number on a number line, STAAR test 3rd grade 2014 © TEA[1]

The first two problems on the second level will be more advanced since in sixth and seventh grade, students have to correctly locate rational numbers on a number line; and by eighth grade, the type of numbers to be placed are irrational. The task here is harder since students cannot use the counting algorithm once used to locate natural numbers to locate rational as well as irrational numbers (Behr, Post, 1984). The number's magnitude is not often whole; therefore, a student has to understand the measurement of each rational and irrational number in order to correctly place the number on the number line (Steffe, Olive, 2010).

On the advanced level, students have to identify the whole interval of number(s) on the number line as the domain or range of a given function; the two questions that relate to the number line used here in this advanced level are similar to the ones given in the algebra 1 EOC exam in 2015.

**16** What is the range of the function graphed on the grid?




**F** $\{x \mid x = -2, 2, 4\}$

**G** $\{y \mid -3 \le y \le 4\}$

**H** $\{x \mid -2 \le x \le 4\}$

**J** $\{y \mid y = -3, 0, 4\}$

Figure 2-3 Identify the range of the given function, EOC Algebra I 2015 © TEA[1]

The second aspect of number sense and the real number system that will be assessed is the ability to compare numbers. According to TEKS 2C a fourth grade student is expected to compare and order whole numbers to 1,000,000,000 and represent comparisons using the symbols >, <, or = (TEKS subchapter A, 2012). The mathematics TEKS in middle school that relates to this TEKS is TEKS 3B of grade sixth; a student is expected to determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one. Thus, in middle school the number comparison concept gets a little more complicated since students cannot just simply compare the number; the students have to identify an increase or decrease a given quantity via an operation, which needs more work than just the comparison. For instance, when increasing a number, students must know that the new number will be larger compared to the one they started out with; moreover, students must also figure out which number to multiply or even divide by the original to get to the larger number. Starting in sixth grade, as listed in TEKS 2A, students will be introduced to other sets in the real number system which means a student is required to be able to recognize the relationship between these sets of numbers. In both cases, elementary level and middle school level, students need to be able to compare, decompose, or evaluate numbers.

**4. Arrange the following real numbers from smallest to largest.**

0.45 0.444... 0.454454445... 0.454545... 0.455

Figure 2-4 A question from the KAAP project, real number module, a suitable question to use in the intermediate level assessment[2]

The ability to compare numbers, especially rational numbers will be extremely useful once students learn about linear function, linear equations and linear inequalities in high school; rate of change or the slope of linear equation is often represented in reduced rational form. Being able to compare and evaluate different rates of change will also follow student through several mathematics courses in high school.

TEKS 6A-6G in algebra 1, TEKS 7B in geometry, and TEKS 1D in pre-calculus all require that students be able to analyze the slope of functions. For instance, in calculus using the changes in slope of the tangent line of a function, students can tell how the original function behaves. Moreover, how the slope of the tangent line changes also reveals the shape of the function. The ability to compare numbers will be reflected in questions three and four of each level in the assessment.

**8** The slope of the line that passes through the points $(-6, w)$ and $(-10, 4)$ is $\frac{1}{8}$. What is the value of $w$?

**F** 36

**G** 34

**H** $\frac{9}{2}$

**J** $\frac{1}{2}$

Figure 2-5 Slope related, EOC Algebra I 2015 © TEA[1]

The third component that will be assessed in this assessment is the ability to do arithmetic with different numbers. This is the most obvious element of number sense and the real number system. One cannot claim that they have number sense if they cannot do number calculation. There will be two questions in each level of this type. According to fourth grade TEKS 4A, the student is expected to add and subtract whole numbers and decimals to the hundredths place using the standard algorithm (TEKS Subchapter A, 2012). In middle school, a six grader is required to be able to add, subtract, multiply, and divide integers as well as multiply and divide rational numbers fluently (grade 6 TEKS 3D, 3E, 2012); once they get to high school, student still need to be able to do calculation; however, at this point, student will not only have to add, subtract, multiply or divide real numbers, they need to be able to do all the calculations with algebraic expressions also. According to Algebra 1 TEKS 10A and 10B (2012), the student is expected to add, subtract and multiply polynomials of degree one and degree two.

The ability to compute with rational numbers also shows up again in Algebra 2 once students learn about rational function; students must be able to solve rational equations that have real solutions as listed in Algebra 2 TEKS 6 I; and students must be able to add, subtract, multiply and divide rational expressions in order to solve rational equation. If students can do calculations with rational numbers well, then they can also do the same with rational expressions (Teaching Mathematics through Problem Solving). Questions five and six on each level of the assessment will be used to assess a student on his or her ability to do arithmetic with different numbers.

**19** Which expression is equivalent to $2m\left(\frac{3}{2}m + 1\right) + 3\left(\frac{5}{3}m - 2\right)$?

**A** $3m^2 + 5m - 1$

**B** $\frac{3}{4}m^2 + \frac{23}{9}m - 6$

**C** $3m^2 + 7m - 6$

**D** $\frac{3}{4}m^2 + \frac{5}{9}m - 1$

Figure 2-6 Evaluate algebraic expression, EOC Algebra 1 2015 © TEA[1]

The next element of number sense is the ability to decompose each given number or change a given number into different forms. This aspect was mentioned as “Form of a Number” in *The Components of Number Sense an Instructional Model* for Teachers by Faulkner. According to Valerie Faulkner, students will develop a sense of number equality; students will find out that they can change a number to different forms while maintaining the value of that number (Faulkner, 2009). It is really helpful for each student to develop this skill since he or she will need to break numbers down to smaller components many times while in a math class. In third grade, as stated in TEKS 2A (2012), a student is expected to compose and decompose numbers up to 100,000 as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate; similarly, a fourth grader must be able to represent a fraction a/b as a sum of unit fractions 1/b, where a and b are whole numbers and b > 0, including when a > b (grade 4, TEKS 3A, 2012).

**12** What is the prime factorization of 196?

**F** $2^2 \cdot 7^2$

**G** $2 \cdot 7^2$

**H** $2 \cdot 7 \cdot 14$

**J** $2^2 \cdot 49$

Figure 2-7 Prime factorization, STAAR test 6th grade 2013 © TEA[1]

In middle school, students will have to add, subtract, multiply and divide rational numbers fluently (seventh grade, TEKS 3A, 2012); in order to work well with rational numbers, students have to be able to factor the denominator of each given rational number into a prime number or at least into different factors; subsequently, factoring is a form of number decomposition, and students must be able to factor to find the common denominator. Thus a middle school student will not be successful when working with rational numbers if he or she cannot factor.

**53** Which expression is equivalent to $-6x^2 - 11x - 4$?

**A** $(3x + 7)(3x - 3)$

**B** $(-3x + 4)(2x - 1)$

**C** $(3x - 7)(3x + 3)$

**D** $(-3x - 4)(2x + 1)$

Figure 2-8 Factoring trinomial, EOC Algebra 1 2015 © TEA[1]

Number decomposition in high school is similar in concept; however, the type of "number" that a student has to work with has changed. Besides real numbers, student will also have to factor algebraic expressions and polynomials; for instance, Algebra 1 TEKS 10E states, students are expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two. Moreover, according to Algebra 1 TEKS11A, the student is expected to simplify numerical radical expressions involving square roots. These two TEKS both relate to different forms of numbers, in which student can break down or rewrite the given number in other forms. According to Valerie Faulkner, factoring or decomposing numbers would become a habit that follows a student over the year (Faulkner, 2009). Question seven and eight in each assessment level will cover number decomposition.

The last component of the number sense and the real number system on which students will be assessed is the ability to interpret or discriminate different solutions given real life situations. This aspect will be assessed on the last two questions of each level of the assessment. Student should develop the skill to recognize why a certain number can be a solution while others may not. TEKS 3A required a fifth grader to estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division; while grade seven TEKS 1B requires a student to use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution (TEKS Subchapter B, 2012). The requirement for students to analyze solutions presents often in high school TEKS. For instance, in Algebra 2 TEKS 4G (2012) requires students to identify extraneous solutions of square root equations, and TEKS 5E (2012) wants student to determine the reasonableness of a solution to a logarithmic equation. This means a student has to be able to pick and choose the right solution from different numbers or even sets of numbers.

**8** Renting video games from Store S costs $2.50 per game plus a monthly fee of $5.00. Renting video games from Store T costs $5.00 per game with no monthly fee. The monthly cost to rent video games depends on the number of video games, *v*, rented. Which inequality represents the situation when the monthly cost at Store S is less than the monthly cost at Store T?

**A** $2.5v + 5 < 5v$

**B** $2.5v + 5 > 5v$

**C** $7.5v < 5v$

**D** $7.5v > 5v$

Figure 2-9 Analyze and choose the most reasonable solution, STAAR test 8th grade 2015 © TEA[1]

This assessment will be used to measure students' understanding of the real number system and number sense. The complete assessment will be presented on the next page.

Table 2-1: Level 1 Real Number Knowledge Assessment

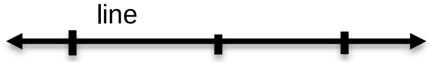
| **Level 1** (7 or more correct go to next level) | | **Score** | |
|---|---|---|---|
| 1. Locate 108 on the number line  100 110 120 | 2. Which number does point S best represent?  31 71 S 260<br>A. 61 B. 100<br>C. 191 D. 91 | | |
| 3. What number is closer to 0<br>a. $\frac{2}{3}$<br>b. 1 | 4. Compare the folowing<br>$\frac{5}{4}$ □ $\frac{7}{6}$ | | |
| 5. 5 x $\frac{9}{4}$ = | 6. 12 ÷ $\frac{1}{4}$ = | | |
| 7. Which of the following has the same value as 7,093?<br>A. 7,000 + 200 + 3<br>B. 7,000 + 100 + 9<br>C. 7,000 +90 + 3 | 8. Complete the following<br>$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \_\_ + \_\_ + \_\_$ | | |
| 9. Which of these is closest to the width of a student's chair?<br>A.15 feet B 15 yards<br>C. 15 miles D. 15 inches | 10. What is a reasonable height of an adult<br>a. 42 in<br>b. 72 in<br>c. 92 in | | |

Table 2-2: Level 2 Real Number Knowledge Assessment

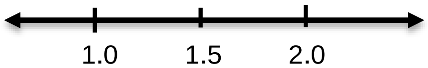
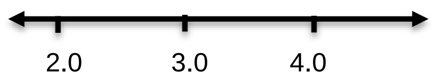
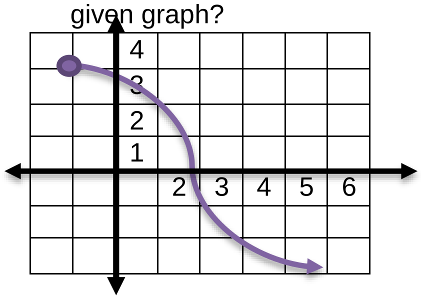
| **Level 2** (5 or more correct go to next level) | | **Score** | |
|---|---|---|---|
| 1. Locate 1.48 on the number line   | 2. Locate π on the number line   | | |
| 3. Compare the following $5 \times \frac{4}{5} \square\ 5$ | 4. Compare the following $4\frac{11}{3} \div \frac{3}{4} \square\ 4\frac{11}{3}$ | | |
| 5. $\frac{3}{7} \times \frac{28}{9} =$ | 6. $8.25 \div \frac{2}{5} =$ | | |
| 7. Fill in the blank $3.\overline{6} = 3 + \text{—}$ | 8. What is the prime factorization of 196? * | | |
| 9. There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?<br>A. 110 B. 173<br>C. 100 D. 163 ** | 10. A cylindrical barrel has a diameter of 19.875 inches. Which of the following is the best estimate of the circumference of the barrel in feet?<br>A. 10 ft B. 2 ft<br>C. 5 ft D. 1 ft ** | | |

Table 2-3: Level 3 Real Number Knowledge Assessment

| **Level 3** (7 or more correct) | | **Score** | |
|---|---|---|---|
| 1. What is the domain of the given graph?<br> | 2. What is the range of the graph given in problem 1? | | |
| 3. Which inequality is equivalent to<br>−3x + 2y > 5y + 9<br><br>A. y > x + 3 B. y > −x − 3<br>C. y < x − 3 D. y < −x – 3*** | 4. Which of the following describes all the solutions to the inequality 5x + 7y ≥ 22 when y = −4?<br><br>A. x ≤ 10 B. x ≤ −10<br>C. x ≥ 10 D. x ≥ −10*** | | |
| 5. For what value of c will the graphs of $y = 2x^2 - 36x + c$ and $y = 2(x - 9)^2 - 18$ be the same? (5) | 6. What value of p makes the equation below true?<br>$\frac{19}{4p-1} = 5$ (5) | | |
| 7. The slope of the line that passes through the points (−6, w) and (−10, 4) is $\frac{1}{8}$.<br>What is the value of w? | 8. A rectangular prism has a width of x inches, a length of $x^2y$ inches, and a height of $y^2$ inches. What is the volume in cubic inches of this rectangular prism? (4) | | |
| 9. If a is a rational number and b is irrational. Discuss whether a + b is irrational or rational; justifying any claims by using examples to illustrate (6) | 10. The sum of a number, n, and its square root can be represented by the equation $y = n + \sqrt{n}$ . If y = 20, which of the following is true<br><br>A. n = 16 B. n = 4<br>C. n = 16 and n = 25<br>D. n = 4 and n = 5 (5) | | |

* Question 12, released Mathematics STAAR test 6th grade 2013 © TEA[1]

** Question 4 and 19, released Mathematics STAAR test 6th grade 2014 © TEA[1]

*** Question 25 and 37, Algebra 1 EOC 2014 © TEA[1]

(4) Question 8 and 12, Algebra 1 EOC 2015 © TEA[1]

(5) Question 2, 9 and 13, Algebra 2 EOC 2013 © TEA[1]

(6) KAAP project Real Number module[2]

# Chapter 3

## Methods

The assessment was used to assess 124 high school students, ages 16-18, enrolled in different high school math courses. In order to evaluate the secondary students' number sense and knowledge of the real numbers system, the number sense assessment tool was used. Knowledge of real number system and number sense would be demonstrated through the ability to compute, locate, compare, interpret, and decompose different numbers in different situations (Number: what is there to know?, 2001). Students' numbers sense and knowledge of the real numbers system would be divided into three different levels: basic, intermediate, and advance.  Some high school students understood the number system up to their grade level while others may just be at the intermediate or basic level. Another aspect taken into consideration while gathering data from the assessment was that students did not have to master the lower level perfectly in order to move on to the next one. However, students must know at least seventy percent of each level to be considered passing that level.

Students who participated in this study were all enrolled in a suburban high school in the southwestern United States; all students have either enrolled in or completed Algebra II. Students were not prepared on any topic of number sense prior to being assessed. The assessment was given to students during a normal period at the end of the first quarter of the school year. The researcher graded this number sense assessment from level one to level three; there was no partial credit given, and each correct question was equivalent to a ten percent grade point. Students had to get at least seventy percent correct from each level to be considered as passing. Students were not allowed to use calculators nor discuss with their classmate. This assessment is a simple pencil-and-paper test. Students were encouraged to solve the problems in the number sense assessment using number sense; the use of arithmetic was not encouraged. Students were also given a time limit of 30 minutes to complete the assessment. According to Berch (2005), when it comes to number sense, timed tests are important since it may “reveal subtle yet important differences in numerical information processing that may not be tapped by assessing accuracy alone”.

The results of the number sense assessment were collected and analyzed in different ways to find the correlation between data. First, for every student, the correct percentages of each level of the number sense assessment was compared with the percentage of the standardized test score, as well as the first quarter Algebra II grade. Then, another table was created to compare the levels of the standardized test and the number sense assessment test. The three categories for the standardized test were converted into numbers: “Not met” was number one, “Met” was number two, and “Advanced” was number three. The results of the assessment test were also converted to numbers: level one was number one, level two was number two and level three was number three. In both cases, each student was assigned the highest number out of the three. The data was compared and charted. This, in some sense, revealed the relevance of the number sense assessment results.

# Chapter 4

## The Results

The assessment was given to students and the data was collected; since students were enrolled in different high school math courses, their scores for the most recent common standardized test (STAAR) and their grades for the first quarter of the highest common high school math course will be used as part of the data. Out of the 124 students, 82% are enrolled in the same level of high school math course, and 18% are enrolled in another higher math class. Also 82% took the standardized test in 2014, and the other 18% took the test two years earlier. However, the data for their standardized tests should be consistent since both tests were implemented on the same reporting categories. This standardized test is chosen to be used since it has a reporting category that relates to number sense. The reporting category mentioned is Properties and Attributes; there were twelve questions in this category which make up about twenty two percent of this standardized assessment. In the new version of this standardized test which will be given in 2015-2016 school year, the reporting categories Properties and Attributes will be given another name which is Number and Algebraic Methods; this name reflects the number sense component better than the previous one.

The correct percentages of the standardized test and level one Number Sense Assessment will be the first two sets of data to be compared. The data will be plotted using Excel scatter plot; the standardized percentages are the independent x values, and the level I percentages are dependent y values.

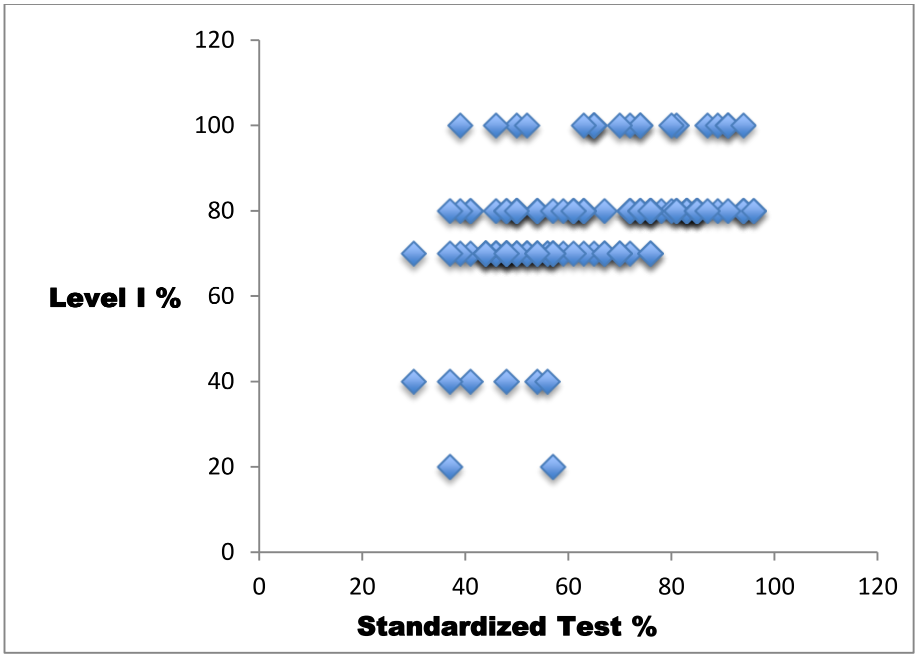



Figure 4-1 Standardized Test vs. Level I

Using the TI 84 Plus calculator with linear regression to find a line of best fit, y = .362x + 54. 231 is obtained as a linear model for the data. The linear regression has a multiple regression $r^2$ of .175, which means only about 18 percent of variation can be predicted by the other variable.

The correlation coefficient r also shows a weak correlation; it is .42, which indicates that there is not much relation between the standardized test score and the level I result. However, the diagnostic ratio is positive which indicated that as the level one percentage is increasing, the standardized test score is also increasing.

The second sets of data to be compared are the standardized test percentages and level II results. The same method used in the first set of data will be used throughout the process.

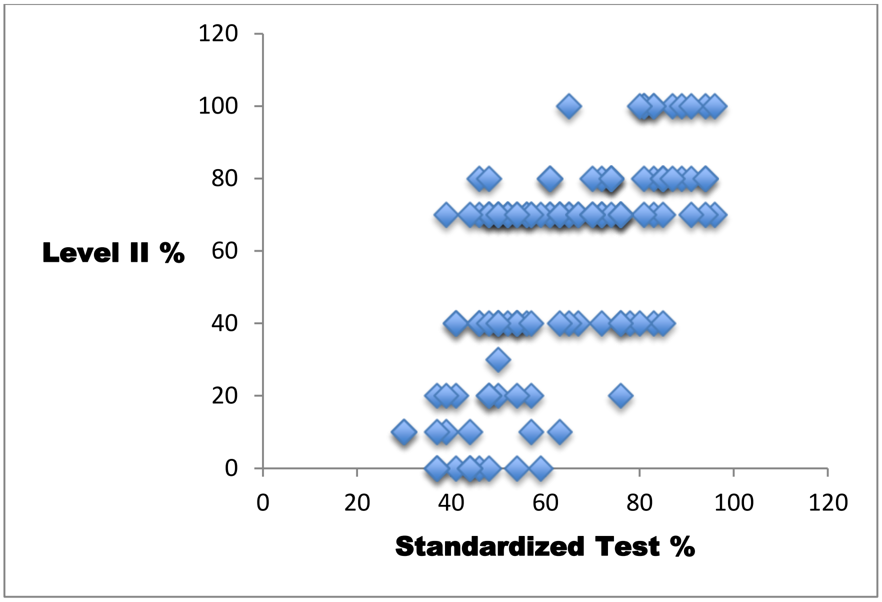



Figure 4-2 Standardized Test vs. Level II

This time the line of best fit is y = 1.101x -13.882. The linear regression has a multiple regression value $r^2$ of .41, which means that 41 percent of the variability of the dependent variable is explained by the independent variable. The correlation coefficient r is .64, which shows a much stronger correlation between the dependent and independent quantity. Also, the correlation coefficient is positive which reflects that the level II number sense assessment result varies directly with the standardized tests.

The next sets of data that are plotted are the level II results of the Number Sense assessment test and the percentages of the standardized test.

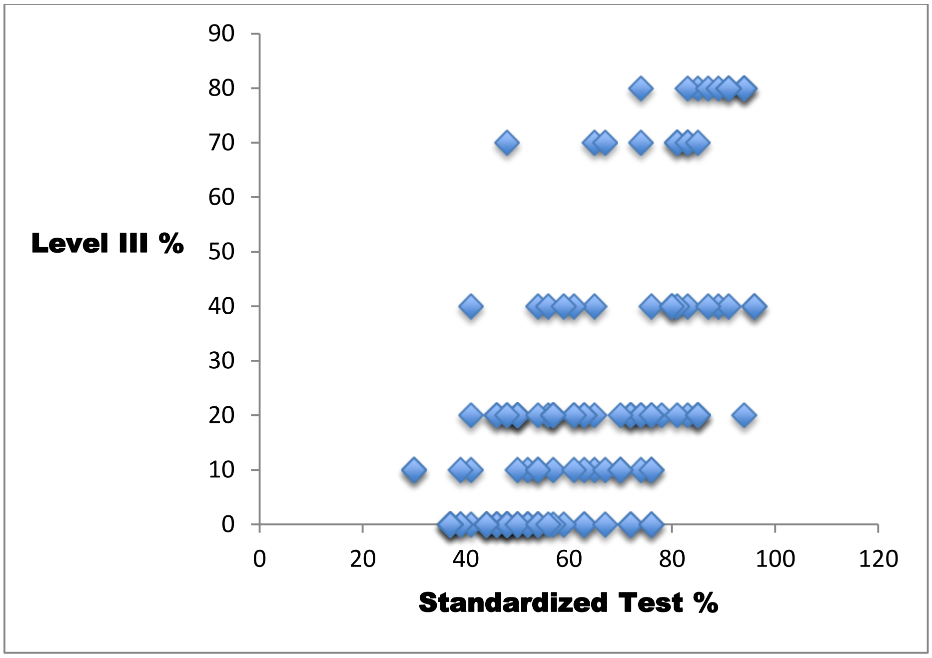



Figure 4-3 Standardized Test vs. Level III

The line best fit for this set of data is y = .952 x -37.431. The linear regression has a multiple regression $r^2$ of .42, which means 42 percent of variation can be predicted by the other variable. The correlation coefficient r is .65, which shows correlation on the stronger side between the standardized test score and the Level III of the number sense result. Also the diagnostic ratio is still positive which reflects that as the level III number sense assessment increase, the standardized test score also increase.

Similar method was used to analyze the result of students' grade for algebra 2 first quarter and the result of the number sense assessment.

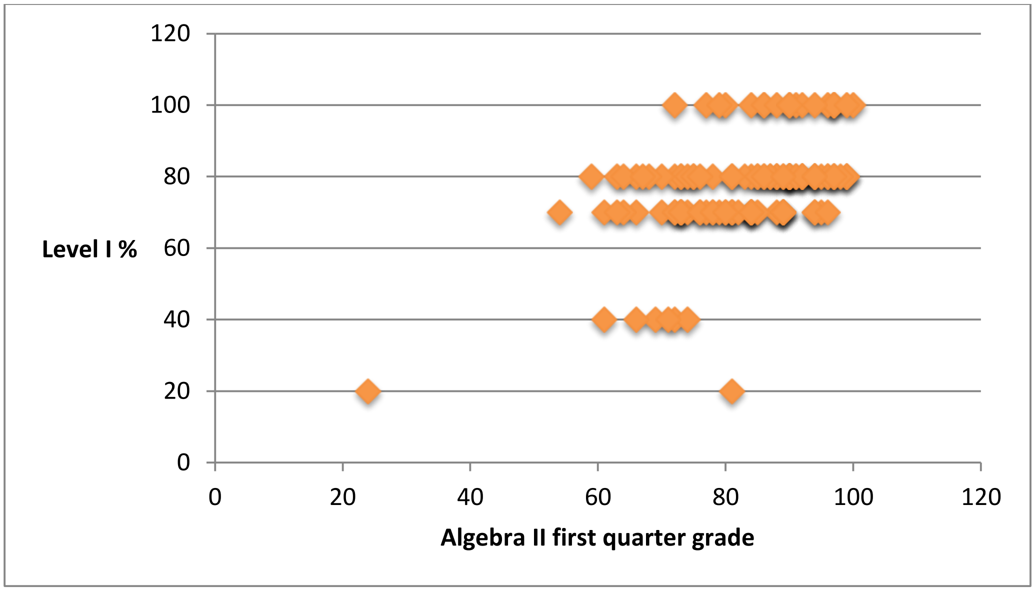



Figure 4-4 Algebra 2 first quarter vs. level I

The linear model that best fit this data is y = .603x +26.260, with the multiple regression $r^2$ is .203 and correlation coefficient r is .45. This result indicates that there is a positive correlation between the first quarter grade and the level I percentage. However, only twenty percent of the data can be predicted by the other variable.

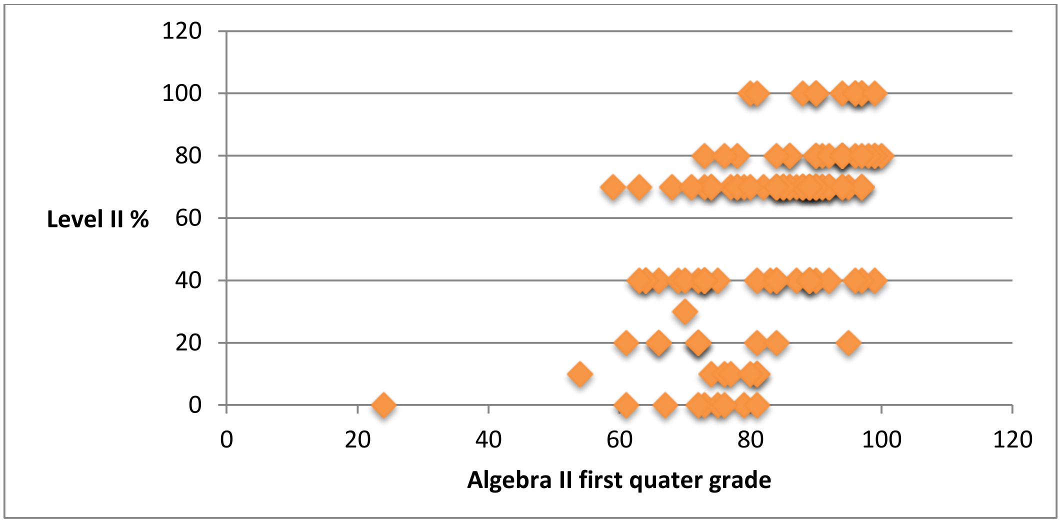



Figure 4-5 Algebra 2 first quarter vs. Level II

For the level II percentage, the linear model that best fit the data is y = 1.355x - 55.866; the multiple regression $r^2$ is .310 which indicates that about 31 percent of the dependent variable can be explained by the independent variable. The correlation coefficient is r = .557 which is the strongest diagnostic ratio in all three sets of data. This shows a positive correlation, which indicated that in the 31% of the data, there is a positive correlation between the level II percentage and the Algebra II first quarter grade.

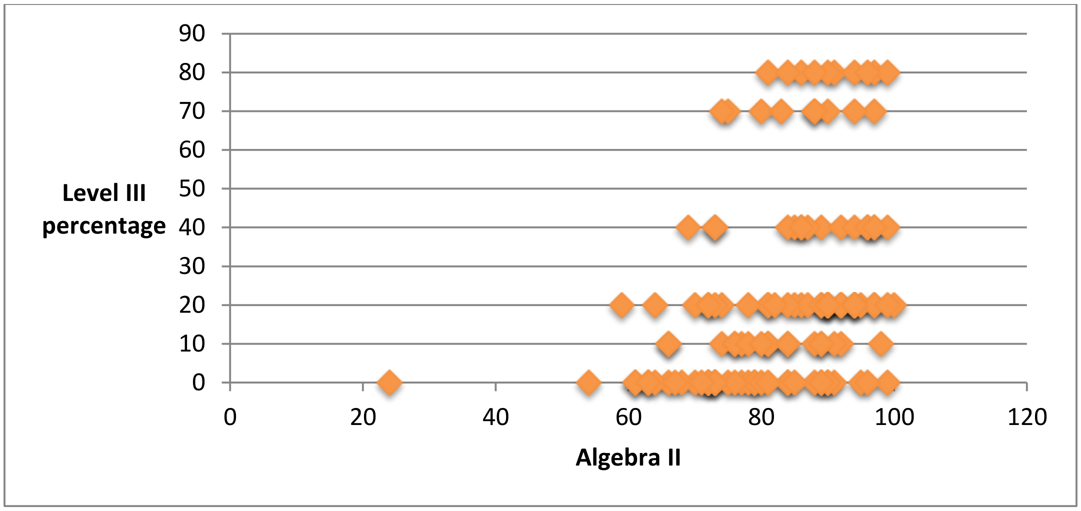



Figure 4-6 Algebra 2 first quarter vs. Level III

The linear model that best fit this set of data is y = .758x -39.228 with the multiple regression $r^2$ is .124 and r correlation coefficient is .352; this is a very weak correlation since only 12 percent of the dependent variable can be predicted by the independent variable.

Out of the first three data comparisons, level II and level III are the ones most likely to reflect students' math abilities in standardized tests; this conclusion was drawn on the second round of comparisons between the number sense assessment result and the first quarter grade of Algebra 2; besides, out of the two levels II and level III, level II data is more consistent with secondary students' mathematics understanding. So in this study, out of the three levels of number sense assessment, level II is the best indicator found when it comes to assess student's high school math ability.

Another table was created to compare the level of the standardized test and the number sense assessment test. The three categories for the standardized test were converted into numbers: "Not met" was number one, "Met" was number two, and

“Advanced” was number three. The results of the assessment test were also converted to numbers: level one was number one, level two was number two and level three was number three. For both cases, each student was assigned the highest number out of the three. The data was compared and charted.

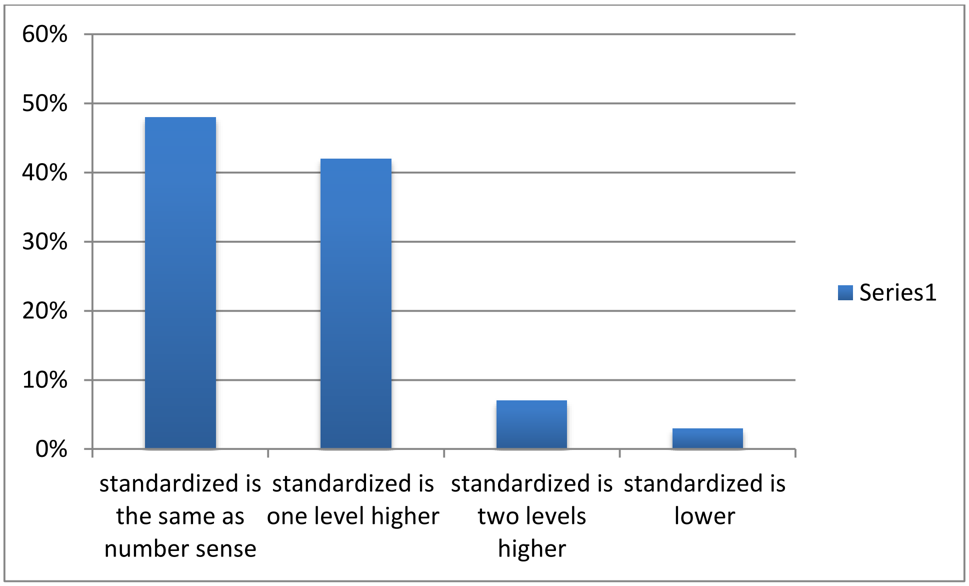



Figure 4-7 Standardized Result vs. Number Sense Assessment Result

When looking at the result between the standardized test and the number sense assessment, about 48% of the result is consistent; this means the result of the standardized test and the number sense assessment are the same. 52 % of the result is inconsistent, with 42% of the result higher by one level on the standardized test result, 7% of the result is higher by two levels on the standardized test result, and 3% with the lower standardized test result.

# Chapter 5

## Discussion

The result of this project reflects that there are some types of relation between secondary students' mathematics performance and their number sense. The number sense assessment developed in this project seems to reveal some aspects of this relation. There will be pros and cons for any assessment; however, to be able to predict the area of number sense that students might struggle in may help teachers in intervention processes. Furthermore, there is a link between students' number sense and their mathematical performance. Better number sense goes hand in hand with better math skills.

Can number sense be taught? According to Gersten, Jordan, and Flojo (2005), if number sense is viewed as a skill or knowledge, then it is teachable (as cited in Berch, 2005). The knowledge of number sense is often taught in the elementary level; however, not all students have the necessary understanding of number sense when reaching secondary level. There are an increasing number of resources and programs created in recent years to boost high school students' knowledge of number sense. In the United Kingdom (UK), the Qualifications and Curriculum Authority has funded *Engaging mathematic of all learners*; a project used by schools in the UK to help link secondary students with number sense using real life applications.

> When referring to the teaching of number sense in the project, Kathotia (2009) stated, to be numerated means much more than to be proficient with arithmetic calculations and algebraic manipulations. It means to have a true sense of what the particular numbers of interest stand for, their magnitude and import being determined by their context, whether they be percentages, rates, averages, measurements, forecast, rankings, targets,…; it means not being confused by a

> clutch of obfuscating figures, being able to efficiently extract a meaningful estimate, and communicate this to others (p. 12).

The above project implied that teachers should link secondary number sense teaching to real life problems in order to spark interest among students. This requires some active planning among teachers so that they can incorporate number sense into their regular lesson plans. Teachers play an important role in teaching students number sense; therefore, they must have a great knowledge about numbers themselves. A recent study done by Sirotic and Zazkis (2004) on the understanding of irrational numbers among pre-service secondary math teachers reflected some troublesome facts. The participants in this study were in their final course in the teacher education program and had at least two calculus courses in their background. Yet, there are a small percentage of these participants who could tell apart rational and irrational numbers. According to Sirotic and Zazkis (2004), one of the reasons for the incorrect responses was the tendency to rely on a calculator for the decimal representation instead of the common fraction representation. There is a need to better inform math teachers about the importance of number sense so that these teachers can prepare their students.

The use of calculators in mathematics classroom has been a topic for debate several times. The important question is: would the usage of calculators improve students' number sense? As a whole, the American educational system since 1975 has warmly embraced the use of calculators in our school system (Banks, 2011, p. 86). Although the use of calculators has been supported by many institutions and organizations (Banks, 2011, p. 84-85), it is mainly for the sole purpose of doing all the complicated computation. Mental math as well as number sense are still crucial and must be taught. It is very important that mental calculations as well as estimation continue to be taught in schools. Such skills are necessary for the mathematical learning process (Pomerantz, 1997, p. 5)

A few of the students who took the Number Sense Assessment had expressed their concerns on the usage of calculator. One common complaint was that the calculator was not allowed. According to one particular student, he would have scored at least 60% on each category if calculators were permitted. It was explained that the purpose of the assessment was to test his number sense ability, and that is why the calculator was not used. He was also informed that he could answer all the questions on the assessment perfectly fine without the aid of the calculator. One particular example usually used while interviewing students was question three of level two.

3. Compare the following

$$5 \times \frac{4}{5} \square 5$$

Figure 5-1: Question 3 of Level 2 Number Sense Assessment

In this question, students do not have to actually calculate the result of each side to compare the two numbers. A student with good number sense would know the left side is smaller compare than the right side; both sides have the same quantity, which is five, and the number five on the left was multiplied with a fraction smaller than one; this will result in a number smaller than five, implying that the left side is smaller. Calculators should only serve as an aid in the teaching and learning of number sense. A calculator cannot replace the intuitive thinking that students should master when it comes to number sense.

On the whole, number sense should be taught across grade levels; student will never be too old or too young to learn number sense.

> According to Halberda, Wilmer, Naiman, and Germine (2012), there is a population trend that suggests the precision of one's number sense improves throughout the school-age years. However, there are also very large individual differences in number sense precision among people of the same age, and most importantly these differences relate to school mathematical performance throughout adolescence and the adult years. The large individual differences and prolonged development of number sense, paired with its consistent and specific link to mathematics ability across the age span, hold promise for the impact of educational interventions that target the number sense.

## Limitations of the Study

First, the sample of students who were studied in the project was limited. Most of the students had similar background, went to the same school, and took the same classes. And all of them were either juniors or seniors from the same local area. Freshmen and sophomores were not considered in the study.

Second, when students took the number sense assessment, they must know all the vocabulary associated with the questions. This means students were not only assessed on number sense, but also mathematics vocabulary.

Third, it was possible for students to correctly answer some questions without using number sense. Students can use procedural understanding to solve some problems on the number sense assessment.

The other limitation for this project was the restricted time period; all of the study had been completed in less than five months. Thus the data set that represented students' secondary math ability was pulled from the first quarter of a school year. A longer time period would illustrate students' mathematics ability better.

## Suggestions for Future Research

Future research on the relationship between number sense and secondary students' mathematics ability would benefit many high school students. Often, number sense is an articulated focus in elementary level mathematics, but number sense is present in mathematics at all levels; furthermore, high school students who are struggling with mathematics are also having problem with number sense. Therefore, the teaching of number sense should be extended to secondary level.

The data gathered for this study could also be examined with a finer grain, paying attention to not just overall performance on the number sense assessment, but also to identifying which components of number sense are strongest and weakest for the subjects.

This project may provide the foundation for other research: when can high school students be assessed on number sense? What type of assessment of number sense can be used for secondary students? To what extend does number sense affect students' mathematic ability? What is the role of technology in developing number sense? What is the threshold of number sense students need to make in order to be success in secondary mathematics? And what kind of number sense teaching and intervention are beneficial for high school students?

Appendix A

Data Tables

Table A-1: Standardized Test and Number Sense Assessment Results

| | Standardized Score | | | | Current | Number Sense Assessment | | | |
|---|---|---|---|---|---|---|---|---|---|
| Raw score | Score | % Score | Met | Advanced | math grade | Level I % | Level II % | Level III % | Result |
| 27 | 3750 | 50% | Yes | No | 7 7 | 100 | 70 | 0 | II |
| 27 | 3750 | 50% | Yes | No | 8 1 | 70 | 20 | 20 | I |
| 29 | 3819 | 54% | Yes | No | 6 3 | 80 | 70 | 0 | II |
| 29 | 3819 | 54% | Yes | No | 8 7 | 80 | 40 | 20 | I |
| 22 | 3568 | 41% | Yes | No | 6 6 | 80 | 20 | 0 | I |
| 30 | 3855 | 56% | Yes | No | 8 9 | 70 | 70 | 40 | II |
| 31 | 3891 | 57% | Yes | No | 8 1 | 20 | 10 | 10 | 0 |
| 30 | 3855 | 56% | Yes | No | 7 3 | 70 | 40 | 20 | I |
| 27 | 3750 | 50% | Yes | No | 7 2 | 80 | 40 | 10 | I |
| 25 | 3676 | 46% | Yes | No | 7 9 | 100 | 70 | 40 | II |
| 41 | 4296 | 76% | Yes | No | 8 4 | 80 | 40 | 20 | I |
| 34 | 4000 | 63% | Yes | No | 7 3 | 80 | 70 | 0 | II |
| 35 | 4041 | 65% | Yes | No | 9 7 | 100 | 70 | 40 | II |
| 27 | 3750 | 50% | Yes | No | 7 0 | 80 | 30 | 0 | I |
| 35 | 4041 | 65% | Yes | No | 8 8 | 100 | 70 | 40 | II |
| 25 | 3676 | 46% | Yes | No | 7 3 | 70 | 0 | 20 | I |
| 39 | 4204 | 72% | Yes | No | 9 9 | 80 | 40 | 10 | I |
| 28 | 3783 | 52% | Yes | No | 9 1 | 100 | 70 | 0 | II |
| 36 | 4080 | 67% | Yes | No | 8 3 | 80 | 40 | 0 | I |
| 27 | 3750 | 50% | Yes | No | 6 6 | 70 | 40 | 0 | I |
| 40 | 4249 | 74% | Yes | No | 9 4 | 80 | 80 | 70 | III |
| 26 | 3723 | 48% | Yes | No | 7 4 | 70 | 70 | 0 | II |
| 52 | 5246 | 96% | Yes | Yes | 8 5 | 80 | 70 | 40 | III |
| 40 | 4248 | 74% | Yes | No | 8 6 | 100 | 80 | 80 | III |
| 46 | 4568 | 85% | Yes | Yes | 9 1 | 80 | 80 | 80 | III |
| 45 | 4505 | 83% | Yes | Yes | 9 0 | 80 | 100 | 80 | III |
| 51 | 5056 | 94% | Yes | Yes | 8 8 | 80 | 70 | 80 | III |
| 48 | 4718 | 89% | Yes | Yes | 7 3 | 80 | 80 | 40 | III |
| 47 | 4639 | 87% | Yes | Yes | 9 7 | 100 | 100 | 80 | III |
| 45 | 4505 | 83% | Yes | Yes | 8 9 | 80 | 70 | 40 | II |
| 35 | 4044 | 65% | Yes | No | 8 0 | 100 | 100 | 70 | III |
| 45 | 4505 | 83% | Yes | Yes | 8 6 | 80 | 80 | 20 | II |
| 51 | 5056 | 94% | Yes | Yes | 9 4 | 80 | 100 | 80 | III |
| 51 | 5056 | 94% | Yes | Yes | 9 9 | 80 | 80 | 80 | III |
| 45 | 4505 | 83% | Yes | Yes | 7 5 | 80 | 40 | 70 | I |

Table A-1—*Continued*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 48 | 4718 | 89% | Yes | Yes | 9 6 | 100 | 100 | 80 | III |
| 49 | 4810 | 91% | Yes | Yes | 8 1 | 80 | 100 | 80 | III |
| 49 | 4810 | 91% | Yes | Yes | 8 4 | 100 | 80 | 80 | III |
| 44 | 4446 | 81% | Yes | Yes | 9 7 | 100 | 100 | 70 | III |
| 52 | 5246 | 96% | Yes | Yes | 9 6 | 80 | 100 | 40 | II |
| 49 | 4810 | 91% | Yes | Yes | 8 6 | 100 | 70 | 40 | II |
| 44 | 4446 | 81% | Yes | Yes | 8 8 | 80 | 100 | 70 | III |
| 45 | 4505 | 83% | Yes | Yes | 9 0 | 80 | 100 | 70 | III |
| 46 | 4568 | 85% | Yes | Yes | 8 8 | 80 | 70 | 70 | III |
| 39 | 4217 | 72% | Yes | No | 8 5 | 80 | 70 | 20 | II |
| 46 | 4588 | 85% | Yes | Yes | 9 0 | 80 | 40 | 20 | I |
| 33 | 3971 | 61% | Yes | No | 7 4 | 80 | 70 | 20 | II |
| 28 | 3783 | 52% | Yes | No | 8 9 | 70 | 40 | 0 | I |
| 25 | 3676 | 46% | Yes | No | 9 0 | 80 | 80 | 20 | II |
| 22 | 3568 | 41% | Yes | No | 7 5 | 80 | 0 | 0 | I |
| 26 | 3712 | 48% | Yes | No | 5 9 | 80 | 70 | 20 | II |
| 42 | 4333 | 78% | Yes | Yes | 9 7 | 80 | 40 | 20 | I |
| 26 | 3712 | 48% | Yes | No | 7 2 | 40 | 0 | 0 | 0 |
| 25 | 3676 | 46% | Yes | No | 7 3 | 70 | 40 | 20 | I |
| 27 | 3750 | 50% | Yes | No | 8 7 | 80 | 70 | 20 | II |
| 35 | 4041 | 65% | Yes | No | 8 9 | 70 | 40 | 20 | I |
| 33 | 3965 | 61% | Yes | No | 8 6 | 80 | 70 | 40 | I |
| 29 | 3819 | 54% | Yes | No | 6 4 | 80 | 40 | 20 | I |
| 28 | 3783 | 52% | Yes | No | 8 4 | 70 | 70 | 10 | II |
| 28 | 3783 | 52% | Yes | No | 8 8 | 70 | 70 | 0 | II |
| 31 | 3891 | 57% | Yes | No | 7 2 | 70 | 20 | 20 | I |
| 43 | 4396 | 80% | Yes | Yes | 9 2 | 80 | 40 | 40 | I |
| 22 | 3568 | 41% | Yes | No | 6 9 | 40 | 40 | 40 | 0 |
| 22 | 3568 | 41% | Yes | No | 8 1 | 70 | 40 | 20 | I |
| 26 | 3712 | 48% | Yes | No | 9 5 | 80 | 20 | 0 | I |
| 39 | 4204 | 72% | Yes | No | 9 0 | 100 | 80 | 20 | II |
| 39 | 4204 | 72% | Yes | No | 9 0 | 80 | 70 | 0 | II |
| 25 | 3676 | 46% | Yes | No | 7 0 | 70 | 40 | 0 | I |
| 39 | 4204 | 72% | Yes | No | 9 5 | 70 | 70 | 20 | II |
| 31 | 3891 | 57% | Yes | No | 9 2 | 80 | 70 | 20 | II |
| 51 | 5069 | 94% | Yes | Yes | 100 | 100 | 80 | 20 | II |
| 24 | 3640 | 44% | Yes | No | 6 1 | 70 | 0 | 0 | I |
| 26 | 3712 | 48% | Yes | No | 6 4 | 70 | 40 | 0 | I |

Table A-1—*Continued*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 32 | 3928 | 59% | Yes | No | 9 4 | 70 | 70 | 40 | II |
| 20 | 3500 | 37% | Yes | No | 6 1 | 40 | 20 | 0 | I |
| 44 | 4451 | 81% | Yes | Yes | 9 6 | 80 | 80 | 40 | II |
| 26 | 3712 | 48% | Yes | No | 7 8 | 70 | 70 | 0 | I |
| 24 | 3640 | 44% | Yes | No | 7 9 | 70 | 0 | 0 | I |
| 41 | 4296 | 76% | Yes | No | 9 2 | 80 | 70 | 10 | II |
| 21 | 3531 | 39% | Yes | No | 6 8 | 80 | 70 | 0 | II |
| 43 | 4396 | 80% | Yes | Yes | 9 9 | 100 | 100 | 40 | II |
| 21 | 3531 | 39% | Yes | No | 7 2 | 100 | 20 | 0 | I |
| 26 | 3712 | 48% | Yes | No | 8 4 | 70 | 20 | 0 | I |
| 40 | 4249 | 74% | Yes | No | 9 2 | 100 | 80 | 20 | II |
| 34 | 4000 | 63% | Yes | No | 9 0 | 80 | 70 | 0 | II |
| 20 | 3500 | 37% | Yes | No | 6 7 | 80 | 0 | 0 | I |
| 44 | 4464 | 81% | Yes | Yes | 9 0 | 80 | 70 | 20 | II |
| 27 | 3750 | 50% | Yes | No | 9 1 | 80 | 70 | 10 | II |
| 16 | 3334 | 30% | No | No | 7 4 | 40 | 10 | 10 | 0 |
| 21 | 3531 | 39% | Yes | No | 7 6 | 70 | 10 | 10 | I |
| 41 | 4296 | 76% | Yes | No | 7 2 | 70 | 20 | 0 | I |
| 41 | 4296 | 76% | Yes | No | 9 2 | 80 | 70 | 20 | II |
| 34 | 4000 | 63% | Yes | No | 9 0 | 100 | 70 | 20 | II |
| 36 | 4080 | 67% | Yes | No | 8 4 | 70 | 70 | 0 | II |
| 16 | 3334 | 30% | No | No | 7 7 | 70 | 10 | 10 | I |
| 29 | 3819 | 54% | Yes | No | 6 6 | 40 | 20 | 10 | 0 |
| 40 | 4249 | 74% | Yes | No | 9 7 | 80 | 70 | 20 | II |
| 46 | 4575 | 85% | Yes | Yes | 7 8 | 80 | 80 | 20 | II |
| 32 | 3928 | 59% | Yes | No | 7 6 | 80 | 0 | 0 | I |
| 34 | 4000 | 63% | Yes | No | 8 4 | 70 | 40 | 20 | I |
| 20 | 3500 | 37% | Yes | No | 5 4 | 70 | 10 | 0 | I |
| 46 | 4575 | 85% | Yes | No | 9 9 | 80 | 80 | 20 | II |
| 34 | 4000 | 63% | Yes | No | 8 1 | 80 | 10 | 10 | I |
| 29 | 3831 | 54% | Yes | No | 7 3 | 70 | 40 | 0 | I |
| 27 | 3750 | 50% | Yes | No | 6 3 | 70 | 40 | 0 | I |
| 41 | 4296 | 76% | Yes | No | 8 9 | 80 | 70 | 0 | II |
| 38 | 4161 | 70% | Yes | No | 9 4 | 100 | 80 | 20 | II |
| 40 | 4249 | 74% | Yes | No | 9 8 | 80 | 80 | 10 | II |
| 33 | 3965 | 61% | Yes | No | 7 6 | 70 | 80 | 10 | II |
| 24 | 3640 | 44% | Yes | No | 8 5 | 70 | 70 | 0 | II |
| 24 | 3640 | 44% | Yes | No | 8 0 | 70 | 10 | 0 | I |

Table A-1—*Continued*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 33 | 3965 | 61% | Yes | No | 9 4 | 80 | 80 | 20 | II |
| 26 | 3712 | 48% | Yes | No | 9 4 | 70 | 80 | 20 | II |
| 29 | 3819 | 54% | Yes | No | 8 1 | 70 | 0 | 0 | I |
| 36 | 4080 | 67% | Yes | No | 8 9 | 70 | 70 | 10 | II |
| 31 | 3891 | 57% | Yes | No | 9 6 | 70 | 40 | 0 | I |
| 18 | 3334 | 37% | No | No | 2 4 | 20 | 0 | 0 | 0 |
| 47 | 4647 | 87% | Yes | Yes | 9 7 | 80 | 80 | 40 | II |
| 38 | 4161 | 70% | Yes | No | 7 8 | 70 | 70 | 10 | I |
| 41 | 4296 | 76% | Yes | No | 9 4 | 80 | 70 | 20 | II |
| 30 | 3855 | 56% | Yes | No | 7 1 | 40 | 70 | 0 | 0 |
| 31 | 3891 | 57% | Yes | No | 8 2 | 70 | 70 | 20 | II |
| 29 | 3819 | 54% | Yes | No | 8 4 | 70 | 70 | 10 | II |
| 38 | 4161 | 70% | Yes | No | 8 0 | 70 | 70 | 10 | II |
| 41 | 4296 | 76% | Yes | No | 8 9 | 70 | 40 | 10 | I |

Table A-2: Standardized level and Number Sense Assessment level

| Not meet (1) | Met (2) | Advanced (3) | Level I % | Level II % | Level III % | Result for Standardized test | Result number sense |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 0 | 0 | 0 | 2 | 0 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 9 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 9 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 3 | 2 | 3 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 3 |
| 1 | 2 | 0 | 1 | 2 | 3 | 2 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 0 | 1 | 2 | 3 | 2 | 3 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |

Table A-2—*Continued*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 3 | 3 | 1 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 3 | 1 | 2 | 3 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 3 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 3 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 3 | 1 | 0 | 0 | 3 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 3 | 1 | 0 | 0 | 3 | 1 |
| 1 | 2 | 0 | 0 | 0 | 0 | 2 | 0 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 3 | 1 | 0 | 0 | 3 | 1 |
| 1 | 2 | 0 | 0 | 0 | 0 | 2 | 0 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |

Table A-2—*Continued*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 0 | 0 | 0 | 2 | 1 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 2 | 0 | 0 | 0 | 0 | 2 | 0 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |

Table A-2—*Continued*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 2 | 3 | 1 | 2 | 0 | 3 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 1 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 0 | 2 | 0 | 2 | 0 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 2 | 0 | 2 | 2 |
| 1 | 2 | 0 | 1 | 0 | 0 | 2 | 1 |

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